

Some General Principles:

- Embrace a “Holy Curiosity” of students;
- Motivate, but not drill students;
- Analyze from complexity to simplicity;
- Search for practical applications.

Part 1: Motivational Case Studies in Introductory Math Classes and Clubs

- Restore Digits in Calculations:

$$\begin{array}{r}
 * 1 * * \\
 \times \quad 1 * * \\
 \hline
 * 1 * * \\
 + * * * 1 * \\
 * * * 1 \\
 \hline
 8 * * 4 * *
 \end{array}$$

- Factor Analysis of Large Numbers:

Prove that any six-digit number, which has the same three first and last digits (written in the same order), has factors (divisors) of 7, 11, and 13.

- Find the number of 0's in the expression:

$$1 \times 2 \times 3 \times \dots \times 98 \times 99 \times 100.$$

- What is the last digit of the number $2597^{5927} \pmod{10}$? (Try to calculate 7^N at $N > 18$ with MS Excel™. Why “0”?).

- Explore primes and Modular Arithmetic! Learn more about Galois fields, Euler's totient function, Fermat's Little Theorem...

- Study the RSA Public-Key encryption algorithm and message digital signatures.

- Link to other disciplines: Explore linguistic text properties in deciphering ciphertexts. ■

REFERENCES: [1] Guzdial, M. *Introduction to Computing and Programming in Python: A Multimedia Approach*. Upper Saddle River, NJ: Pearson, Prentice Hall, 2005.

[2] Gill, K. M. Putting Life on Mars, *Rivier Academic Journal*, Vol. 9, No. 1, 2017, pp. 1-27.

Part 2: Math and Computer Graphics

Based on the human perception of light (3 eye cone cells sensitive to red, green, and blue), the RGB color model [1] allocates 24 bits for each pixel to represent $2^{24} = 16,777,216$ different colors. *Jython* programs [1] can be used for manipulating with image colors, creating a negative (Fig.1), reducing red-eye, etc.



Fig. 1: Creating the negative image.

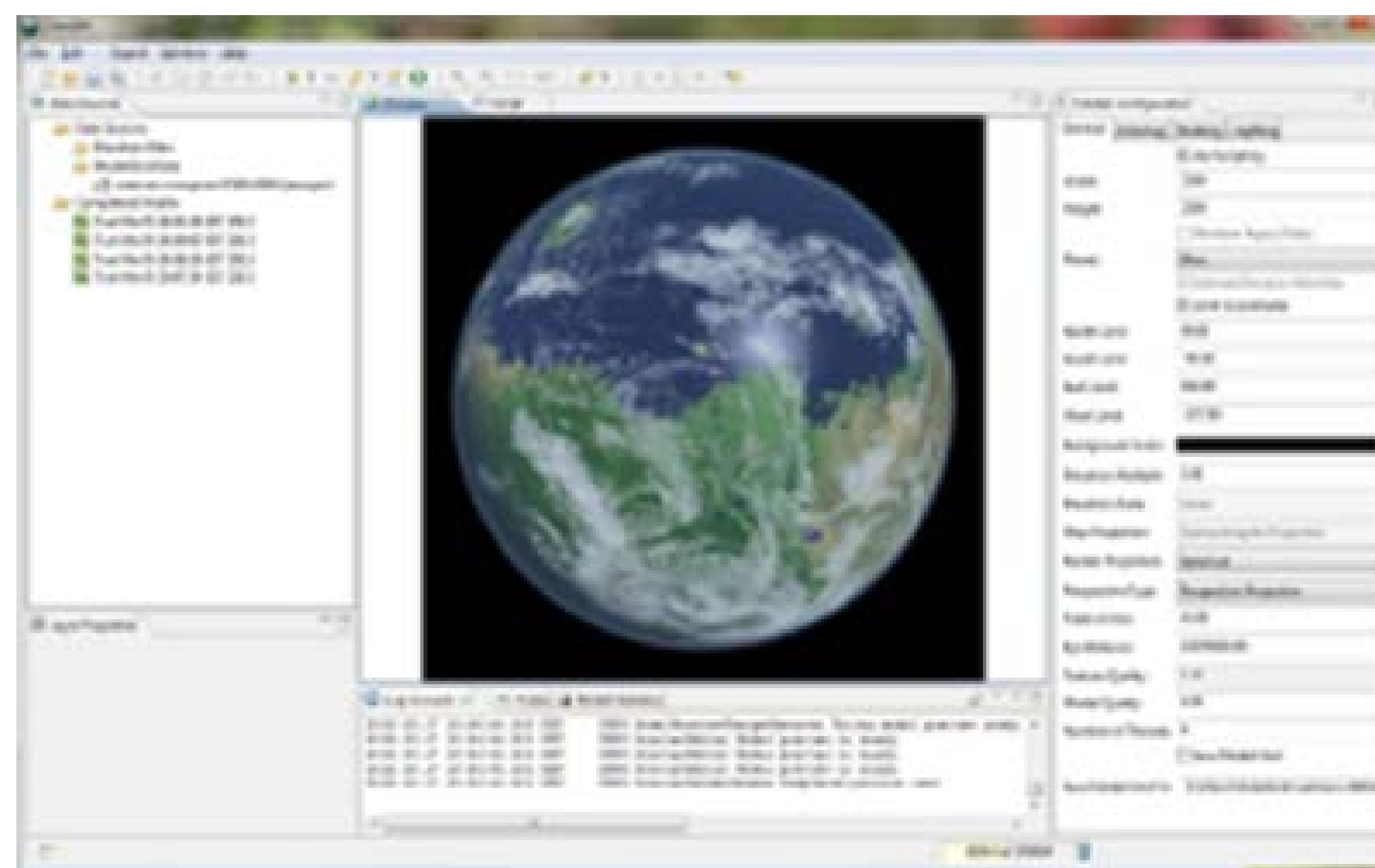


Fig. 2: jDem846 with the Living Mars project. Kevin Gill developed the Living Mars project [2] that included methods of computer graphics and planetary science. With the jDem846 tool, he created a visualization of the Mars (see Fig. 2) as could look with a living biosphere. ■

Part 3: Strange Attractors: an evolution of dynamic systems

Case studies examine numerical modeling of chaotic dynamic systems (e.g., turbulence, weather forecast, and economic system development).

Lorenz's system bifurcations model convection in the Earth's atmosphere:

$$\frac{dx}{dt} = a(y - x) \quad (1)$$

$$\frac{dy}{dt} = x(b - z) - y \quad (2)$$

$$\frac{dz}{dt} = xy - cz \quad (3)$$

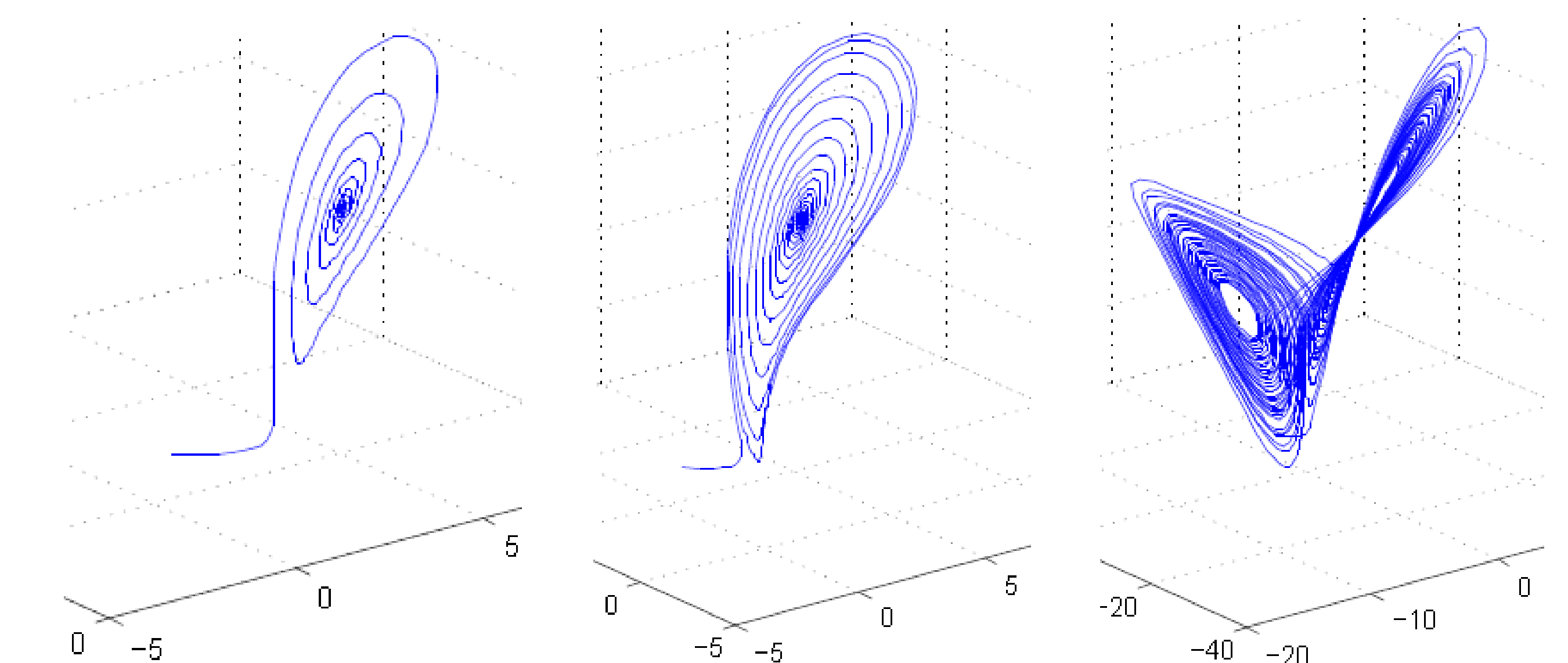
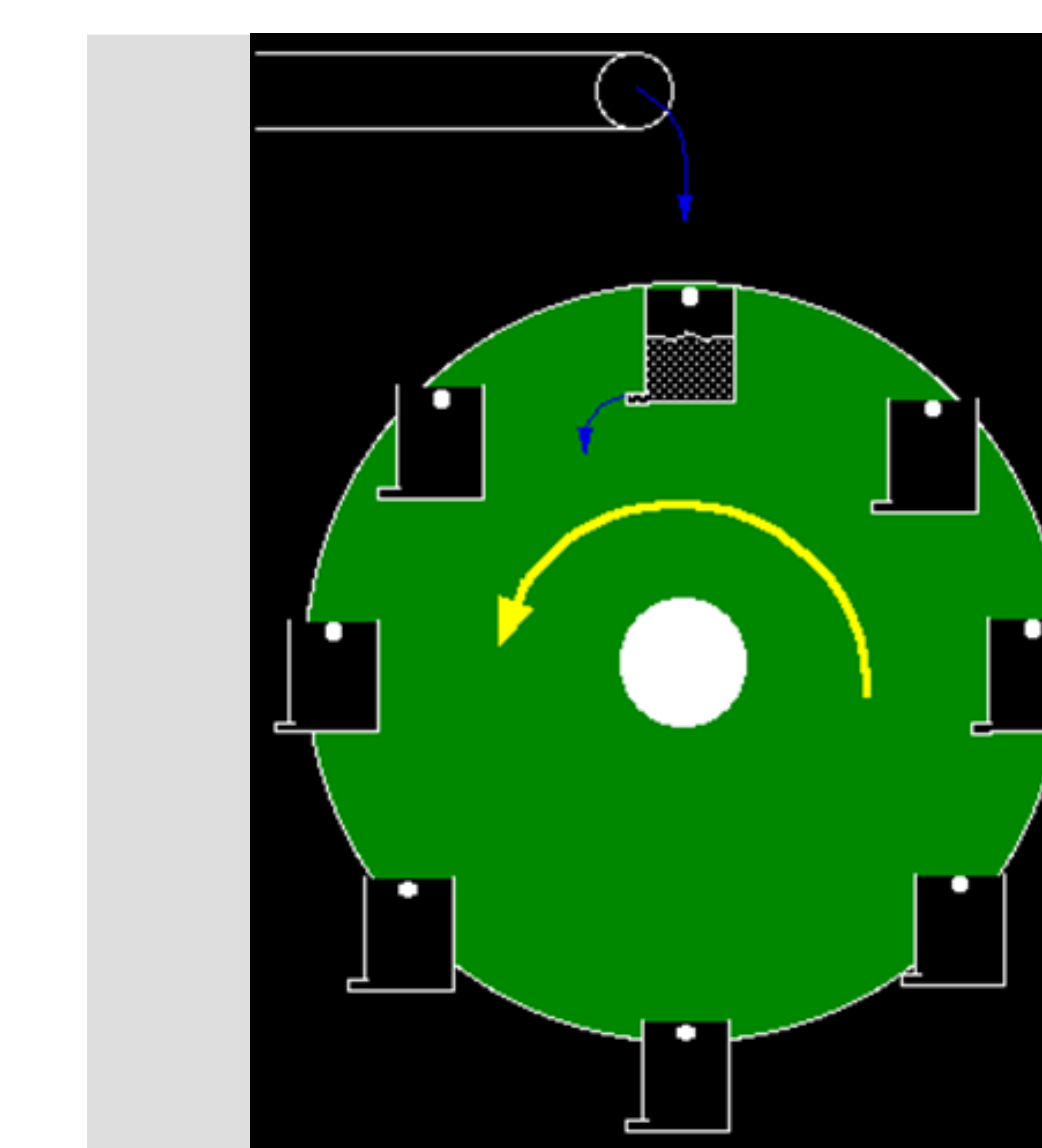


Fig. 3: L-R: Solutions of the Lorenz system (Eqs. 1-3) for different values of the Rayleigh number $b = 12, 16, \text{ and } 28$; $a = 10, c = 8/3$.



The waterwheel (a physical model of the Lorenz's system) was built by P. Paultje for the Dutch Annual Physics Teacher Conference in 2005. ■

