Kronecker, Einstein, and the Cross

Michel Tombroff

109 Avenue Kamerdelle, 1180 Brussels, Belgium; michel@tombroff.com

Abstract

Two eminent scientists, the German mathematician Leopold Kronecker, and the German-born theoretical physicist Albert Einstein, made comments in which they refer to God in their respective rebuttals of the theory of transcendental and transfinite numbers and of the consequences of quantum theory. The apparent contradiction between the objective activity of science and the subjective experience of faith has been the subject of numerous commentaries over the centuries by scientists and philosophers, and a source of inspiration to many artists. In this paper, I present a brief review of this science vs. God dichotomy, starting with Voltaire's "God the watchmaker". I then describe my two recent artworks, *The Necessity of Chance* and *The Work of Man*, inspired by Kronecker's and Einstein's statements. Finally, I explain why I, an atheist, chose the Christian cross as symbol for these artworks.

Science and God

Most scientists eventually have to confront the idea of a divine being, irrespective of their personal attitude with respect to God or religion. Whether a scientist is an atheist, an agnostic or an ignostic, or believes in pantheism or in a Judeo-Christian-Islamic-or-other God, he will eventually, but inevitably, face questions related to the origin of the universe, the first cause, the existence of God, the immortality of the soul, the problem of evil, and the absolute.

To an atheist, this confrontation is generally quite harmless. But to a believer, it may lead to an impasse, especially when initiated by the emergence of new concepts and theories, such as the theory of evolution, the theory of transfinite numbers, quantum physics, big-bang theory or genetics for instance. In such a situation, the dilemma is either to revise one's arguments to the God hypothesis in order to integrate the new theories, or to completely separate the two realms, the scientific and the divine, and decide that the question of God is unreachable to the scientific method.

We could list many examples of intellectuals who confronted this situation. Here are just a few. Voltaire made reference to "God the watchmaker" in his verse *The world embarrasses me and I cannot dream that this watch exists and has no watchmaker.*¹ René Descartes's dualism was heavily influenced not only by science but also by theology, and in his *Meditations Métaphysiques* he proposed a demonstration of the existence of God through an ontological argument ([10], 5th Meditation): in Descartes's argument *the proof of the existence of God proceeds from the existence of an idea of an infinite being in the human mind – an idea of God – to the existence of God himself* [18]. In his *Ethics*, Baruch Spinoza offers us an essentialist proof of the existence of his pantheist divinity: *God, or substance, consisting of infinite attributes, of which each expresses eternal and infinite essentiality, necessarily exists* ([19], Proposition XI).

Blaise Pascal, the prolific French mathematician, physicist, writer and Catholic theologian, made fundamental contributions in many scientific domains (Pascal invented a calculator, the Pascaline; he performed critical experiments to better understand the phenomenon of atmospheric pressure; and he established the foundations of the mathematical theory of probability) and wrote some of the most influential theological texts of all time, the *Pensées* [15], an examination and defense of the Christian Faith,

¹ L'Univers m'embarrasse et je ne puis songer que cette horloge existe et n'ait pas d'horloger [21].

and *Les Provinciales* [16]. In the last letter of *Les Provinciales* Pascal presents a strong argument in favor of the separation of scientific method and faith:

How then do we learn what the facts are? From our eyes, Father, which are the rightful judges of facts, as reason is of natural and intelligible things, and Faith of things supernatural and revealed. But since you compel me to do so, Father, let me tell you that in the opinion of two of the greatest doctors of the Church, Saint Augustine and Saint Thomas, these three principles of knowledge, the senses, reason and Faith each have separate objects, and are certain within their range.²

Closer to our time, eminent scientists with various atheist or religious beliefs wrote or made remarks about God, including Carl Sagan – *The idea that God is an oversized white male with a flowing beard who sits in the sky and tallies the fall of every sparrow is ludicrous. But if by God one means the set of physical laws that govern the universe, then clearly there is such a God* [17] – and Richard Feynman – *Religion is a culture of faith; science is a culture of doubt* [14], to name just two.

In the next two sections, I look more specifically at how two prominent scientists of the 19^{th} and 20^{th} centuries, Leopold Kronecker (mathematician, 1823 - 1891) and Albert Einstein (physicist, 1879 - 1955), made reference to God.

God Created the Integers, Everything Else Is the Work of Man

The second half of the nineteenth century was a very fertile period for mathematics, in particular in the fields of calculus, number theory, and set theory. Several major discoveries, including the completion of the theory of complex and irrational numbers and the proof of existence of transcendental numbers were made by Charles Hermite, Adrien-Marie Legendre, Karl Weierstrass and others. The proof of the transcendence of π in 1882 by Ferdinand Von Lindemann is one of the most notable successes of that period [3].

In parallel, following the early works of Gottlob Frege, Richard Dedekind, Johann Dirichlet and others, Georg Cantor invented set theory and the theory of transfinite numbers [7]. The formidable influence of Cantor's discoveries on all fields of 20th century mathematics led David Hilbert to write, in 1926, that *From the paradise, that Cantor created for us, no-one can expel us.*³

Despite these great successes, the German mathematician Leopold Kronecker, who violently opposed the use of transcendental and transfinite numbers in mathematics, said in 1886 *God created the integers, everything else is the work of man.*⁴ Driven by a strict adherence to finitism and constructivism, and leveraging the strong influence he had on the German and international mathematical scenes, Kronecker repeatedly tried to prevent or delay the publications of Cantor's works.

Kronecker, who came from a Jewish family, kept that religion until a year before his death, when he converted to Christianism. But his reference to God as the creator of the integers tells us less about his religious beliefs or his modest Platonism than about his absolute conviction that mathematics should only deal with finite entities and that proofs and mathematical constructions should only use finite numbers of steps.

² D'où apprendrons-nous donc la vérité des faits ? Ce sera des yeux, mon Père, qui en sont les légitimes juges, comme la raison l'est des choses naturelles et intelligibles, et la foi des choses surnaturelles et révélées. Car, puisque vous m'y obligez, mon Père, je vous dirai que, selon les sentiments de deux des plus grands Docteurs de l'Eglise, saint Augustin et saint Thomas, ces trois principes de nos connaissances, les sens, la raison et la foi, ont chacun leurs objets séparés, et leur certitude dans cette étendue ([16], 18th Letter).

³ Aus dem Paradies, das Cantor uns geschaffen, soll uns niemand vertreiben können [12].

⁴ Die ganzen Zahlen har der liebe Gott gemacht, alles andere ist Menschenwerk [9].

God Does Not Play Dice With the Universe

Half a century later, during the early years of the twentieth century, the founding principles of quantum physics – the part of physics that describes nature at its smallest scale, i.e. at the level of atoms and other subatomic particles – emerged from the pioneering works of Max Planck and Albert Einstein. Then, from the 1920s, the theory was further developed and formalized by Erwin Schrödinger, Werner Heisenberg, Max Born, Niels Bohr and others. Quantum physics describes a very different model of nature compared to the one of classical mechanics discovered by Isaac Newton and Gottfried Wilhelm Leibniz in the 17th century. While classical mechanics' equations describe the world as fully deterministic and causal, quantum theory is based on non-deterministic and probabilistic principles that are, still to this day, difficult to interpret.⁵

Despite the formidable capacity of quantum theory to predict the results of physics experiments, Albert Einstein, one of its early inventors, wrote in a 1926 letter to Max Born that God does not play dice with the $universe^{6}$, expressing his refusal to accept the indeterministic nature of quantum physics.

Einstein's strict determinism and his fascination for Baruch Spinoza's pantheism are well known ([6], [23]). Einstein referred to himself as an agnostic rather than an atheist ([23], p. 340), a religious nonbeliever (ibid., p. 390). But like Kronecker forty years before him who referred to God to deny the existence of transcendental and transfinite numbers, Einstein's reference to God to express his rejection of quantum physics is less a trace of his religious beliefs than the symptom of his malaise to accept the implications of quantum physics.

The Work of Man, the Necessity of Chance

When Kronecker wrote *God created the integers, everything else is the work of man*, and Einstein *God does not play dice with the universe*, they both connected their scientific convictions (constructivism and finitism for Kronecker, determinism and causality for Einstein) to a divine being. Kronecker's is a positive assertion (*God created the integers ...*), while Einstein's is a negative statement (*God does not play dice ...*).

I find these two statements quite interesting, for several reasons. First, because of their great simplicity. Second, because they assert, implicitly but without hesitation, the existence of a God: their assertive syntax (there is no "I think that" or "I believe that" in these statements) tells us that Kronecker and Einstein had no doubt that a God exists. And third, because these statements depict a very anthropomorphic God: for Kronecker, an artisan creating the integers, companion to the mathematician crafting up more complex objects (e.g. complex numbers, transcendental numbers, groups, etc.), and for Einstein, a gambler throwing the dice at the crap table.

These men, their respective passion (mathematics, physics), and their statements inspired me. I therefore decided to conceive two artworks, as an homage to them: *The Necessity of Chance*, and *The Work of Man*, depicted on Figure 1 below.

The Necessity of Chance is made up of a wood panel (90 cm width x 129 cm height), acrylic paint, wood sticks each with a base of 1 cm x 1 cm and a height of 1 cm to 6 cm. Each stick represents the result of the throw of a dice, i.e. 1, 2, 3, 4, 5 or 6. The dice was thrown 6,338 times, represented by so many wood sticks. The obtained pattern is random, except for a cross-shaped region made up of 3s in the middle.

The Work of Man is made up of a wood panel (90 cm width x 129 cm height), acrylic paint, wood sticks each with a base of 1cm x 1cm and a height of 1 cm to 9 cm. The sticks represent the integers, starting at 0 (represented by an absence of stick) up to 1,913 (represented by 4 sticks, i.e. the digits 1, 9, 1 and 3). The work exhibits a certain regular and diagonal pattern, due to the arrangement of the numbers in rows of a

⁵ Richard Feynman famously wrote *I think I can safely say that nobody understands quantum mechanics* [11].

⁶ Jedenfalls bin ich überzeugt, dass der Alte nicht würfelt [5].

fixed size. Some digits are represented by black sticks, which together form the shape of a cross. There are 5,907 sticks in total.



Figure 1. The Necessity of Chance (left) and The Work of Man (right).

Why the Christian Cross?

My position regarding the existence (or not) of God sits somewhere in the region where atheism, agnosticism and pantheism meet – I have to admit that I am still a bit uncertain about the whole affair. What I do know is that I do not adhere to any religion nor do I believe in an anthropomorphic God. So, why did I decide to use the Christian cross as the symbol for my artwork? I could have decided to use another figure: the star of David of Judaism, the crescent moon of Islam, or Hinduism's Aum, for instance.

The choice of the Christian cross was not influenced by religious, aesthetic or cultural considerations, no. It was motivated by the reading of Alain Badiou's *Being and Event* [1], and in particular *Meditation Twenty One*, entitled *Pascal*.

Alain Badiou – Being and Event

It is impossible to summarize Badiou's theory in a few lines, and I refer the reader to the dialogue between Badiou and Gilles Haéri in [2] for an informal introduction, as well as to the excellent synthesis of Badiou's theory by Burhanuddin Baki in [4]. Basically, Badiou declares that *mathematics is ontology and that ontology is mathematics* ([4], p. 248). *Mathematics*, Badiou writes, *is the science of being-as-being, that is,*

ontology, that is, the independent study of all the possible forms of multiplicity as such, of all multiplicities, and therefore of all that is – for all that is, is in any case a multiplicity ([1], p. 78).

Why did this theory have any influence on the choice of the Christian cross as a symbol of divinity in my artworks? In order to explain this, I proceed in three steps below: first, I summarize Badiou's theory of event and truth; second, I describe Badiou's argument – inspired by Lacan – that amongst all religions, Christianity is the one that comes closest to the question of the truth, not just a contemplative truth, but truth based on intervention, and how Blaise Pascal's genius was to recognize this; finally, I argue that Kronecker's and Einstein's statements are similar interventions and, for that reason, that the cross is the right symbol.

Truth: the Result of Militant Activism from an Event

Starting from his platonic and audacious proposition (mathematics = ontology), Badiou develops the conceptualization of the mechanism that leads to the construction of truths in particular situations (artistic, scientific, political, or love). Truth, as opposed to mere "veridicities" which Badiou refers to as encyclopedic knowledge, emerges from the occurrence of an undecidable event, followed by the action of a subject who first names the event and then, patiently and militantly, constructs the fabric of truth around it by deciding which elements of the situation are connected, or not, to the name of the event. Badiou makes an analogy between the notion of "forcing" in set theory⁷ and the way a truth is being constructed and added to a situation by the militant activist (the subject).

Translated into the situation of sciences (in our case, mathematics and physics), the role of the scientist is to militantly propose new, undecidable propositions, in the form of new conjectures and theories: the theory of relativity and of the quanta of energy for Einstein, new developments in number theory and algebra for Kronecker. Both thinkers have been very active militants in their respective fields of activity and have each produced and proved a vast amount of such propositions that have added to the vast collection of human knowledge.

Christianity, Truth and the Cross

In his Pensées, Pascal wrote that The history of the Church ought to be called the history of truth⁸ and in Being and Event Alain Badiou wrote that Lacan used to said that, if no religion were true, Christianity, nevertheless, was the religion which came closest to the question of the truth ([1], p. 212). Badiou interprets Lacan's words to mean that In Christianity, and in it alone it is said that the essence of truth supposes the eventual ultra-one, and that relating to truth is not a matter of contemplation – or immobile knowledge – but of intervention. For at the heart of Christianity there is that event – situated, exemplary – that is the death of the son of God on the cross. By the same token, Badiou adds, belief does not relate centrally to the being-one of God, to his infinite power; its interventional kernel is rather the constitution of the meaning of that death, and the organization of a fidelity to that meaning (ibid.).

Pascal

When faced with the dilemma of confronting his faith with contemporary scientific discoveries, *Pascal's particular genius*, Badiou writes, *lies in his attempt to renovate and maintain the eventual kernel of the Christian conviction under the absolutely modern and unheard of conditions created by the advent of the subject of science ([1], p. 214])*. Badiou then continues: *He [Pascal] illuminated the paradox that at the very moment in which science finally legislated upon nature via demonstration, the Christian God could only remain at the centre of subjective experience if it belonged to an entirely different logic, if the 'proofs of the existence of God' were abandoned, and if the pure eventual force of faith were restituted (ibid.)*.

⁷ The concept of "forcing" was invented by Paul Cohen in 1963 when he proved the independence of the Axiom of Choice and the Continuum Hypothesis [8].

⁸ L'histoire de l'Église doit être proprement appelée l'histoire de la vérité ([15], XLI., p 284).

Kronecker and Einstein

The genius of Pascal, I agree with Badiou, laid in his ability to find a way for his faith and his science to cohabitate, through a militant and faithful intervention linked to Christianity's inaugural event, the death of the son of God on the cross.

I think we can draw an analogy between the genius of Pascal and those of Kronecker and Einstein, based on Badiou's interpretation. Kronecker and Einstein, I argue, followed the same logic – consciously or not – when they made their now famous statements. In *The Necessity of Chance* and *The Work of Man*, I wanted to find a way to represent this connection.

Conclusion

The Necessity of Chance and *The Work of Man* are not statements about God. They are not statements about quantum physics or mathematics. And they are not about Einstein or Kronecker either. They are an attempt to show the trace of the aleatory trajectory of the truth procedure that was inaugurated thousands of years ago, at the cross, by the apostles, then faithfully constructed over the centuries by thinkers such as Pascal, Kronecker, and Einstein. This aleatory trajectory collides, inevitably, with the thoughts and works of scientists, philosophers, artists, theologians and the mere citizens. Today, it touched me.

My atheist conviction has not been affected by this experience. But this project helped me better understand Alain Badiou's theory of truths. So, perhaps I should say that *The Necessity of Chance* and *The Work of Man* are an homage to Alain Badiou.

References

- [1] A. Badiou. *Being and Event*. Bloomsbury Academic. 2013.
- [2] A. Badiou. L'Éloge des Mathématiques. Flammarion, 2015.
- [3] A. Baker. Transcendental Number Theory. Cambridge University press, 1990.
- [4] B. Baki. *Badiou's Being and Event and the Mathematics of Set Theory*. Bloomsbury Academic. 2016.
- [5] M. Born. The Born-Einstein Letters. Palgrave Macmillan. 2004.
- [6] A. Calaprice. The Ultimate Quotable Einstein. Princeton University Press, 2010.
- [7] G. Cantor. *Contributions to the Founding of the Theory of Transfinite Numbers*. The Open Court Publishing Company, 1952.
- [8] P. Cohen. Set Theory and the Coninuum Hypothesis. Dover Publications. 2008.
- [9] J. Dauben. *Georg Cantor. His Mathematics and Philosophy of the Infinite*. Cambridge, MA. Harvard University press, 1979.
- [10] R. Descartes. Méditations Métaphysiques. Collection Folioplus Philosophie, Folio, 2006.
- [11] R. Feynman. The character of physical law. Cambridge, 1967.
- [12] D. Hilbert. "Über das Unendliche". Mathematische Annalen, (95) (1): 161-190, 1926.
- [13] W. Isaacson. Einstein: His Life and Universe. Simon and Schuster, 2008.
- [14] R. Leyton. Surely You're Joking Mr. Feynman. Vintage. 1992.
- [15] B. Pascal. Pensées. Folio, Paris. 2004.
- [16] B. Pascal. Les Provinciales. Gallimard, Paris. 1987.
- [17] C. Sagan. Cosmos. Ballantine Books. 2013.
- [18] A. Schechtman. "Descartes's Argument for the Existence of the Idea of an Infinite Being", in *Journal of the History of Philosophy*, 52 (3):487-517. 2014.
- [19] B. Spinoza. *Ethics*. Cambridge Texts in the History of Philosophy, Cambridge University Press, 2018.

- [20] G. S. Viereck. Glimpses of the Great. Macauley, New York, 1930.
- [21] Voltaire. *Les systèmes et les cabales. Ensuite la Bégueule et Jean qui pleure et qui rit.* Hachette. Paris. 2017.
- [22] H. Weber. Mathematische Annalen, 43, 1-25, 1893.
- [23] M. K. Wisehart (Interview with). "A Close Look at the World's Greatest Thinker." The American Magazine, June 1930.

ANNEX

In this annex I present larger images of The Necessity of Chance and The Work of Man.



Figure 2. The Necessity of Chance



Figure 3. The Work of Man